

# Time-dependent ambulance allocation considering data-driven empirically required coverage

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1st International Workshop on Planning of Emergency Services: Theory and Practice

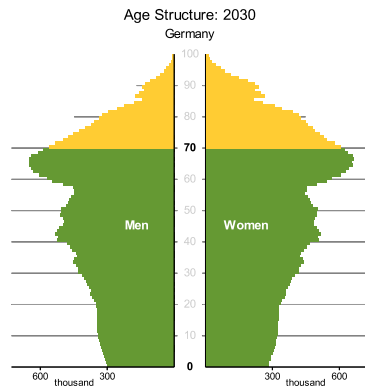
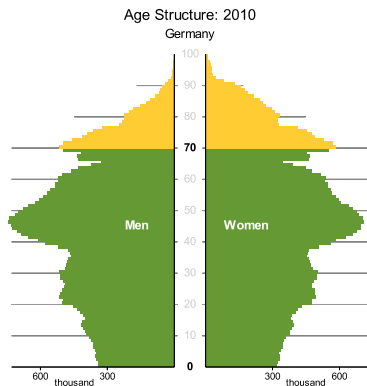
CWI, Amsterdam, 25–27 June 2014

# Outline

1. Motivation
2. Basic idea and models
3. Data driven optimization model for tactical ambulance planning
4. A real world EMS planning problem in the city of Bochum
5. Conclusions and Outlook

# Motivation

## ► Aging population

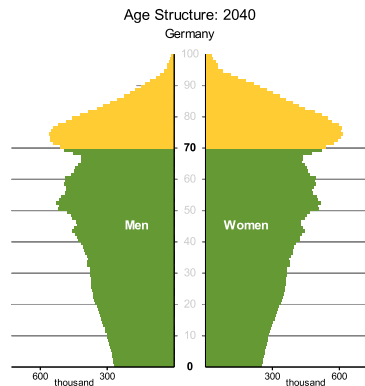
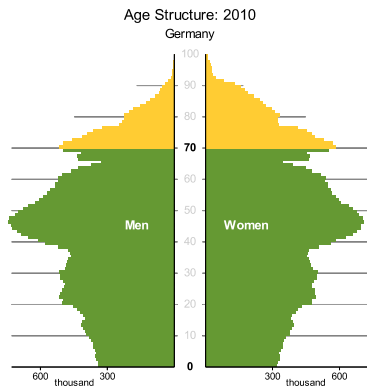


- Age class of 70+ causes 50% of ambulance operations
- EMS demand increases

source: Statistisches Bundesamt 2013

# Motivation

## ► Aging population



- Age class of 70+ causes 50% of ambulance operations
- EMS demand increases

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# Motivation

1. Access to emergency medical services (EMS) is crucial

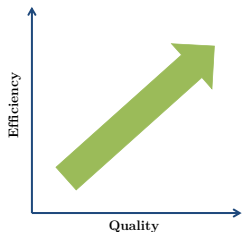
2. Ensuring optimal supply quality

- ▶ Short arrival time
- ▶ Coverage of entire demand area
- ▶ High degree of achievement

⇒ Dependent on location and number of ambulances

3. Efficiency

- ▶ Reduction of fixed costs (avoid overcapacity)
- ▶ Utilization of ambulances



# Quality criteria and objectives

Evaluation of emergency medical services (objective of EMS provider):

- ▶ Degree of achievement (ex post):

$$\frac{\# \text{ operations within a time standard } T}{\# \text{ total number of operations}}$$

Objectives in literature (ex ante):

- ▶ single coverage models
- ▶ double coverage models
- ▶ busy fraction models
- ▶ queue models
- ▶ hypercube models

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1														129				1
2							55	47		92	105	118	130	140	148			2
3						44	56	68	80	93	106	119	131	141				3
4			12	22	33	45	57	70	81	94	107	120	132	142	149			4
5			12	23	34	46	58	70	82	95	108	121	133	143	150	155		5
6	1	5	14	24	35	47	59	71	83	96	109	122	134	144	151	156	161	6
7		6	15	25	36	48	60	72	84	97	110	123	135	145	152	157	162	7
8	3	7	16	26	37	49	61	73	85	98	111	124	136	146	153	158	162	8
9	4	8	17	27	38	50	62	74	86	99	112	125	137	147	154	159	163	9
10		9	18	28	39	51	63	75	87	100	113	126	138					10
11		10	19	29	40	52	64	76	88	101	114	127	139					11
12		11	20	30	41	53	65	77	89	102	115	128						12
13			21	31	42	54	66	78	90	103	116							13
14				32	43	55	67	79	91	104	117							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

# Optimal quality in emergency medical services

Research project: 2 years (work in progress)



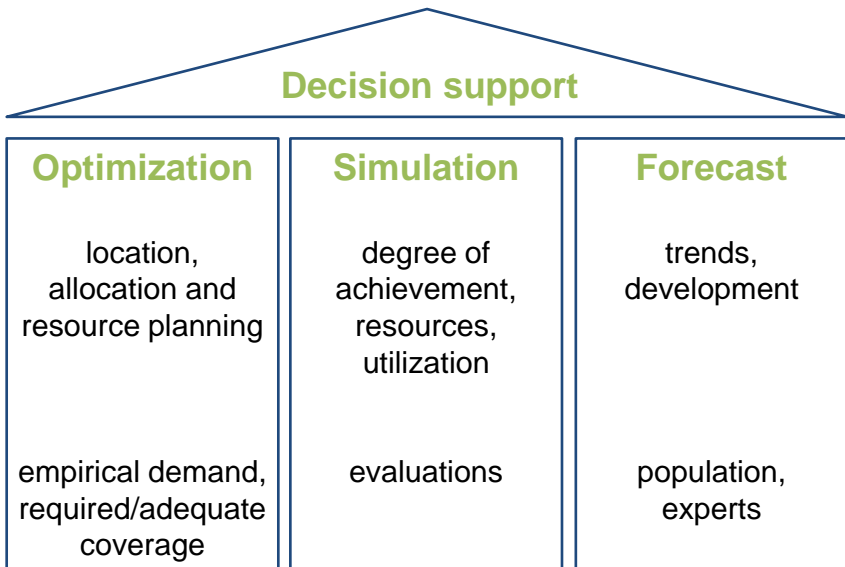
## Focus:

- ▶ Analysis of required resources for EMS
- ▶ Analysis of time-dependent demand and speed fluctuations for EMS
- ▶ Tactical and strategic planning horizon
- ▶ Stochastic influences, uncertain parameters (demand, speed)

## Goal:

- ▶ Dynamic (and robust) optimization model
- ⇒ IT-based decision support tool for local EMS providers

# Decision support tool





# Decision support tool

## Decision support

### Optimization

location,  
allocation and  
resource planning

empirical demand,  
required/adequate  
coverage

### Simulation

degree of  
achievement,  
resources,  
utilization

evaluations

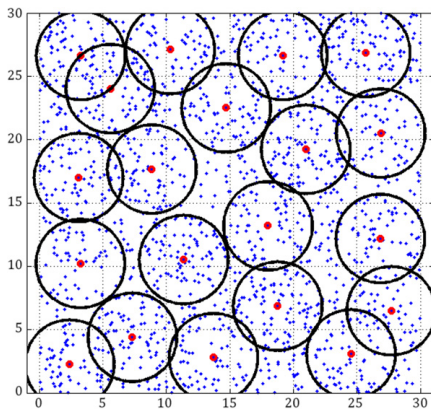
### Forecast

trends,  
development

population,  
experts

## Basic covering location models

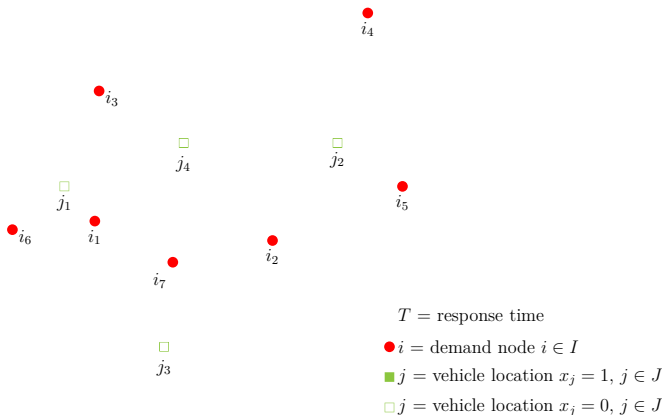
**Idea:** Demand nodes  $i$  have to be covered within a time standard  $T$



source: Zarandi et al. 2011–The large scale maximal covering location problem; page 1565

# Set Covering Problem (SCP)

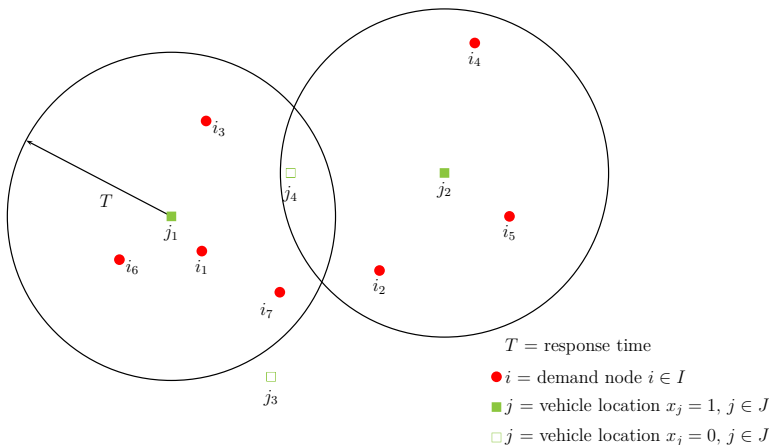
Toregas et al. 1971



$$\mathcal{N}_i := \{j \in J \mid t_{ij} \leq T\}$$

# Set Covering Problem (SCP)

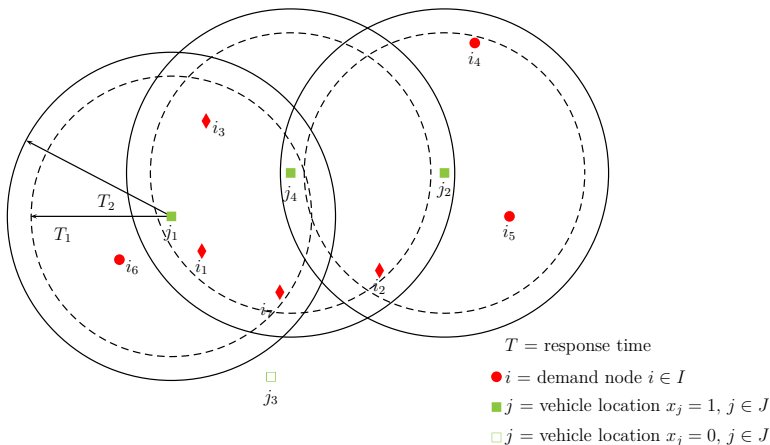
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$$\mathcal{N}_i := \{j \in J \mid t_{ij} \leq T\}$$

# Double Standard Model (DSM)

Gendreau et al. 1997



$$\mathcal{N}_i^{T_\ell} := \{j \in J \mid t_{ij} \leq T_\ell, \ell \in \{1, 2\}\}$$

# Double Standard Model (DSM)

Gendreau et al. 1997

$$\max \sum_{i \in I} d_i x_i^2$$

$$\text{s. t.} \quad \sum_{j \in \mathcal{N}_i^{T_2}} y_j \geq 1 \quad \forall i \in I$$

$$\sum_{i \in I} d_i x_i^1 \geq \alpha \sum_{i \in I} d_i$$

$$x_i^1 \geq x_i^2 \quad \forall i \in I$$

$$\sum_{j \in \mathcal{N}_i^{T_1}} y_j \geq x_i^1 + x_i^2 \quad \forall i \in I$$

$$\sum_{j \in J} y_j = p$$

$$x_i^1, x_i^2 \in \{0, 1\} \quad \forall i \in I$$

$$y_j \in \mathbb{N}_0 \quad \forall j \in J$$

►  $d_i$ : demand at node  $i$

►  $\mathcal{N}_i^{T_\ell} := \{j \in J \mid t_{ij} \leq T_\ell\}$

$$T_1 < T_2$$

►  $p$ : number of ambulances  
(fleet size)

$$x_i^k = \begin{cases} 1, & \text{if demand node } i \text{ is} \\ & \text{covered } k \in \{1, 2\} \text{ times} \\ 0, & \text{else.} \end{cases}$$

$y_j$ : number of ambulances at node  $j$

# Double Standard Model (DSM)

Gendreau et al. 1997 — Limitations: (1) static consideration (2) fixed double coverage

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$$\text{s. t.} \quad \sum_{j \in \mathcal{N}_i^{T_2}} y_j \geq 1 \quad \forall i \in I$$

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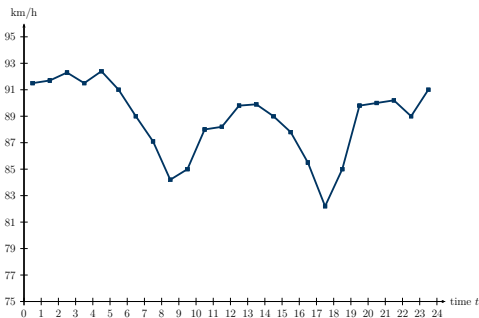
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# Extensions of the Double Standard Model

Time-dependent parameter: speed

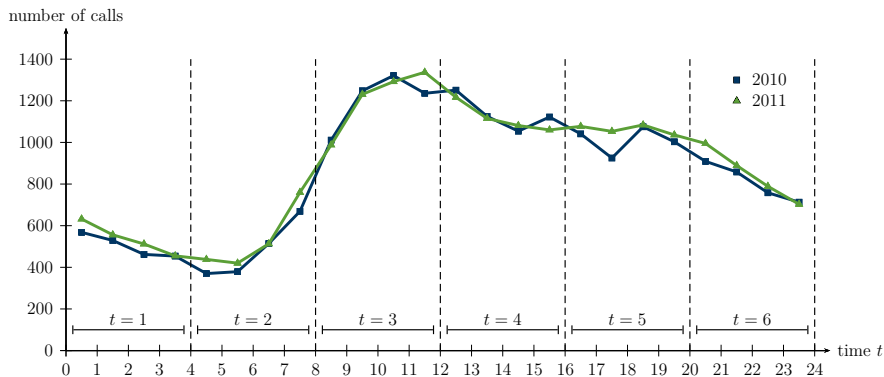


- Time-dependent speed on (city-)motorways (example of the city of Vienna)

Source: Kritzinger et al. (2011)

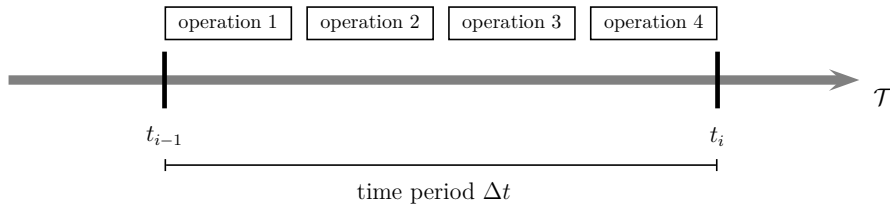


# Time-dependent parameter: demand



⇒ Dynamic considerations required

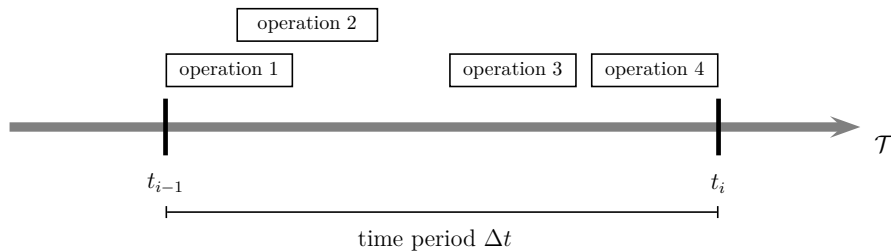
# Time-dependent parameter: degree of required coverage



$\xi$  : number of ambulances,  $X$  : number of parallel operations

$$P(\{X \leq \xi\}) \geq \beta \quad (= 0.95)$$

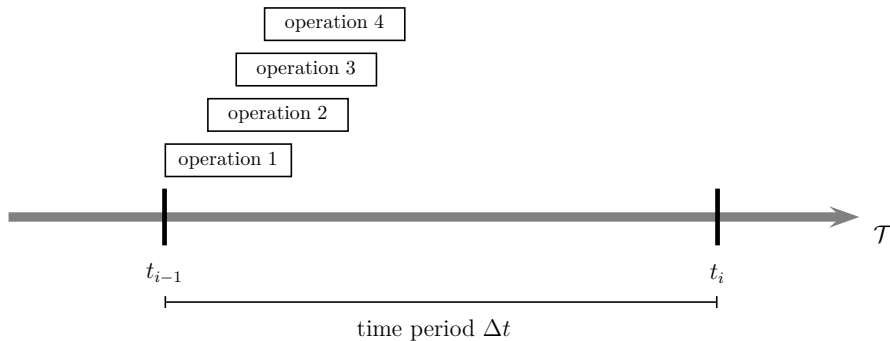
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# Extensions of the Double Standard Model

## Existing:

- ▶ **Speed** is time-dependent and location-dependent
- ▶ Empirical investigation (e.g. Schmid/Doerner 2010; Degel et al. 2014)

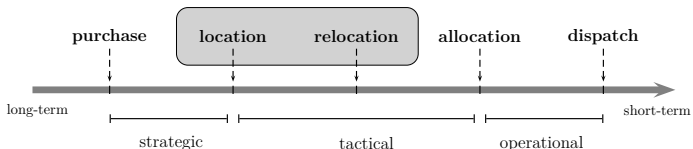
# Extensions of the Double Standard Model

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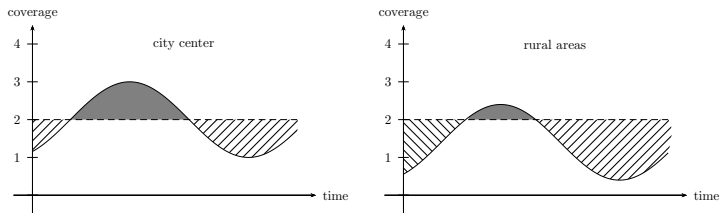
## Additional:

- ▶ Time- and location-dependent **demand**
- ▶ **Empirically required coverage** for each demand node/period
- ▶ **Additional and flexible** ambulance stations
- ▶ **Dynamic** and **flexible allocation** of ambulances to stations



# New optimization approach

## Objectives (1)



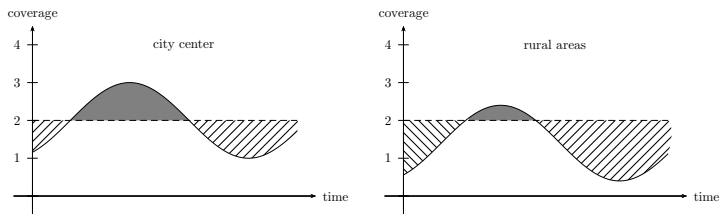
- ▶ Observed: differences between required and actual coverage
- ▶ Maximize the empirically required coverage:

$$x_{it}^k := \begin{cases} 1, & \text{if demand node } i \text{ is covered } k\text{-times in period } t, \\ 0, & \text{otherwise.} \end{cases}$$

- ▶  $k = e(it)$  is determined empirically ( $\leftarrow$  parallel operations)

# New optimization approach

## Objectives (1)



- ▶ Observed: differences between required and actual coverage
- ▶ Maximize the empirically required coverage:  $x_{it}^{e(it)} \in \{0, 1\}$

$$\max \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} d_{it} x_{it}^{e(it)}$$

- ▶  $k = e(it)$  is determined empirically ( $\leftarrow$  parallel operations)



# New optimization approach

## Objectives (2)

- ▶ Relocations due to variations of demand/speed (penalty costs)
- ▶ Utilization of flexible ambulance stations (penalty costs)

$$\min \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} u_{ijt}$$

$$\min \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{F}} y_{jt}$$

$$y_{Dt} = p - p_t \quad \forall t \in \mathcal{T}$$

$$y_{jt} + \sum_{i \in \mathcal{J} \cup \{D\}} u_{ij(t+1)} - \sum_{i \in \mathcal{J} \cup \{D\}} u_{ji(t+1)} = y_{j(t+1)} \quad \forall j \in \mathcal{J} \cup \{D\}, \forall t \in \mathcal{T} \setminus \{T\}$$

$$y_{jT} + \sum_{i \in \mathcal{J} \cup \{D\}} u_{ij1} - \sum_{i \in \mathcal{J} \cup \{D\}} u_{ji1} = y_{j1} \quad \forall j \in \mathcal{J} \cup \{D\}$$

$$u_{ijt} \in \{0, 1\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \cup \{D\}, \forall t \in \mathcal{T}$$

$$y_{jt} \in \mathbb{N}_0 \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$$

$$\vdots$$

# New optimization approach

## Constraints

- ▶ covering constraints
- ▶ location constraints
- ▶ relocation constraints
- ▶ allocation constraints
- ▶ demand constraints

# Bochum

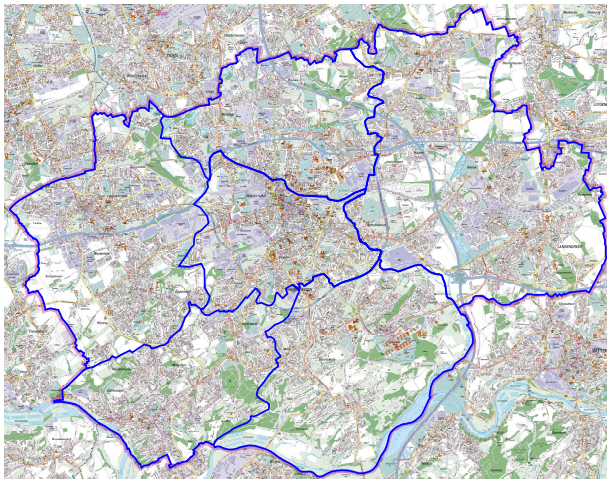


Coordinates

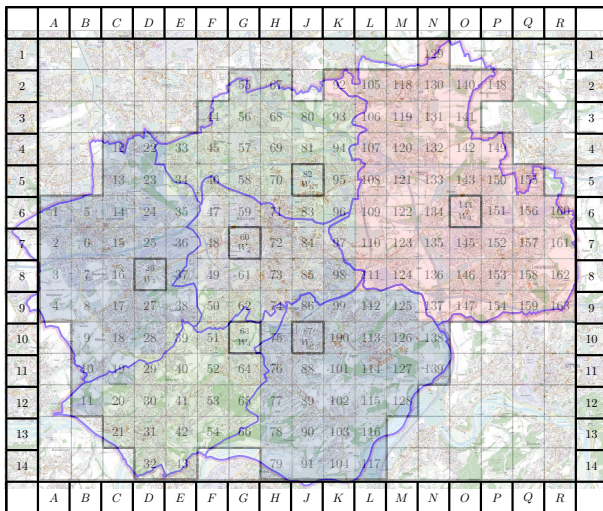
$51^{\circ} 28' 55'' N$   $7^{\circ} 12' 57'' E$

- ▶ 16th biggest city in Germany
- ▶ Area:  $145.4 \text{ km}^2$  (56.1 sq mi)
- ▶ Population: ca. 375,000
- ▶ Population density:  $2,577/\text{km}^2$
- ▶ Services: about 21,000 operations per year
- ▶ Services per 1,000 inhabitants per year: about 56 operations
- ▶ **14** ambulances

# The city area of Bochum



# The city area of Bochum



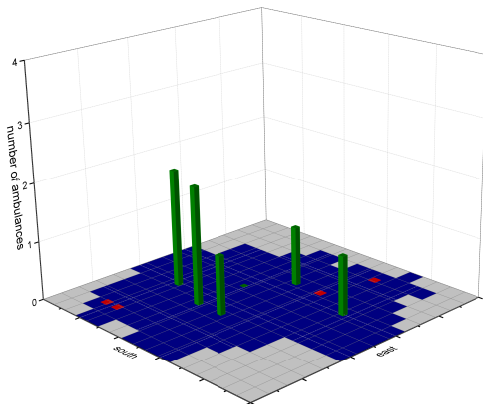
# The city area of Bochum

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													129					1
2							55	67		92	105	118	130	140	148			2
3						44	56	68	80	93	106	119	131	141				3
4			12	22	33	45	57	69	81	94	107	120	132	142	149			4
5			13	23	34	46	58	70	$\frac{82}{W_6}$	95	108	121	133	143	150	155		5
6	1	5	14	24	35	47	59	71	83	96	109	122	134	$\frac{144}{W_3}$	151	156	160	6
7	2	6	15	25	36	48	$\frac{60}{W_2}$	72	84	97	110	123	135	145	152	157	161	7
8	3	7	16	$\frac{24}{W_1}$	37	49	61	73	85	98	111	124	136	146	153	158	162	8
9	4	8	17	27	38	50	62	74	86	99	112	125	137	147	154	159	163	9
10		9	18	28	39	51	$\frac{63}{W_4}$	75	$\frac{87}{W_5}$	100	113	126	138					10
11		10	19	29	40	52	64	76	88	101	114	127	139					11
12		11	20	30	41	53	65	77	89	102	115	128						12
13			21	31	42	54	66	78	90	103	116							13
14				32	43			79	91	104	117							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

# Model results

## Usage of flexible ambulance stations and relocations

Period  $t = 1$ , number of ambulances 7

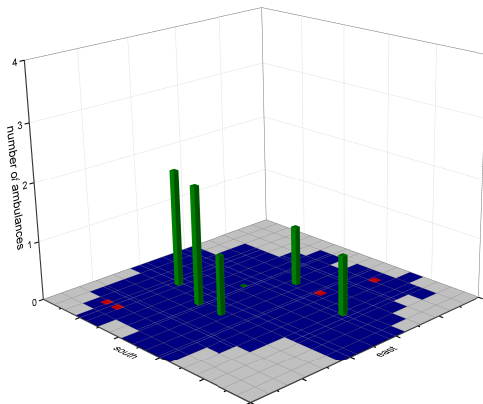


■ main stations, ■ flexible stations

# Model results

## Usage of flexible ambulance stations and relocations

Period  $t = 2$ , number of ambulances 7



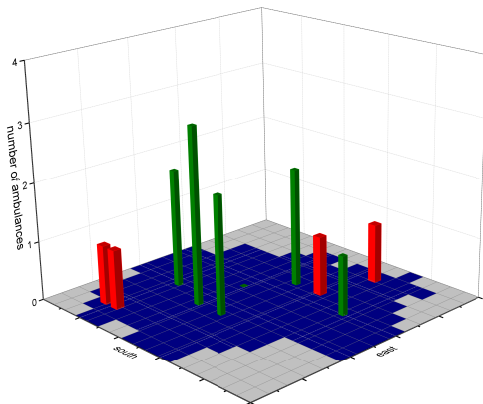
■ main stations, ■ flexible stations



# Model results

Usage of flexible ambulance stations and relocations

Period  $t = 3$ , number of ambulances 14

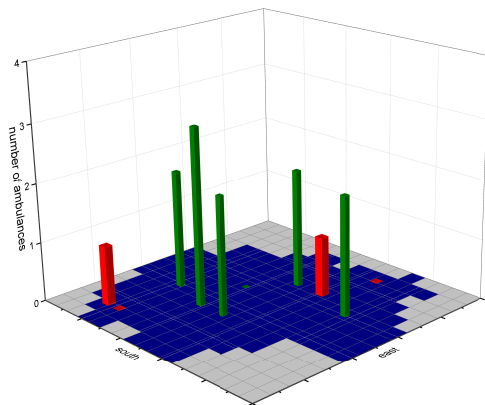


■ main stations, ■ flexible stations

# Model results

Usage of flexible ambulance stations and relocations

Period  $t = 4$ , number of ambulances 13

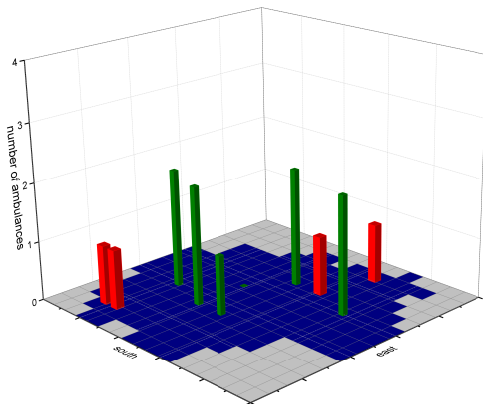


■ main stations, ■ flexible stations

# Model results

Usage of flexible ambulance stations and relocations

Period  $t = 5$ , number of ambulances 13

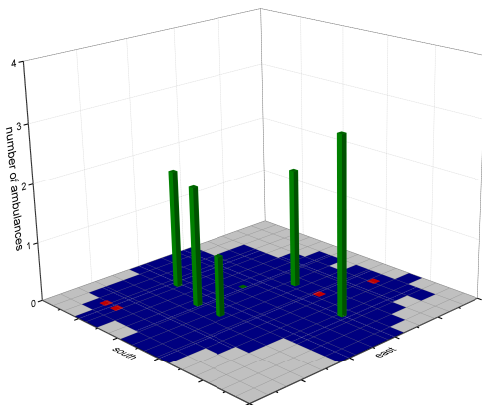


■ main stations, ■ flexible stations

# Model results

Usage of flexible ambulance stations and relocations

Period  $t = 6$ , number of ambulances 10



■ main stations, ■ flexible stations

# Model results

Difference between resulting and required degree of coverage (period 8-12 a.m.)

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													-2					1
2							-2	-1		-1	-2	-2	-2	-2	-2			2
3						-2	-1	-1	-1	-1	-1	-2	2	2				3
4			-2	-2	-2	3	3	3	-1	-1	-1	3	2	2	2			4
5			1	1	5	3	3	3	3	-1	3	3	2	2	2	2		5
6	-2	1	1	5	5	5	2	2	3	3	3	3	2	2	2	2	2	6
7	1	1	1	5	5	5	6	4	3	3	3	2	2	2	2	2	2	7
8	1	1	1	5	6	5	6	4	4	4	-1	2	2	2	2	2	-2	8
9	1	1	1	2	6	7	7	3	4	0	-1	-1	2	2	2	-2	-2	9
10		1	1	2	2	7	4	4	0	0	-1	-1	-2					10
11		-2	1	2	2	0	0	0	0	0	-1	-1	-2					11
12		-2	-2	-2	-1	-1	0	0	0	-1	-1	-2						12
13			-2	-2	-2	-1	-1	0	-1	-1	-2							13
14				-2	-2			-2	-2	-2	-2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

status quo

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													-1					1
2							-2	0		2	0	0	-1	-1	-1			2
3						-2	0	0	2	2	2	0	2	1				3
4			-2	-2	-2	0	0	1	1	2	2	4	2	2	0			4
5			0	0	0	0	0	1	1	1	4	4	2	1	0	0		5
6	-2	0	0	0	0	1	-1	0	1	1	3	3	1	1	0	0	0	6
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9	0	0	0	2	2	4	4	1	2	2	0	0	0	0	0	-2	-2	9
10		1	2	4	3	4	2	2	2	2	0	0	-2					10
11		0	2	4	4	3	2	2	2	2	0	0	-2					11
12		0	0	0	2	2	3	2	2	0	0	-2						12
13			0	0	0	2	1	2	0	0	-2							13
14				0	0			-2	-2	-2	-2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

max double coverage

# Model results

Difference between resulting and required degree of coverage (period 8-12 a.m.)

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													-2					1
2						-2	-1			-1	-2	-2	-2	-2	-2			2
3						-2	-1	-1	-1	-1	-1	-2	2	2				3
4			-2	-2	-2	3	3	3	-1	-1	-1	3	2	2	2			4
5			1	1	5	3	3	3	3	-1	3	3	2	2	2	2		5
6	-2	1	1	5	5	5	2	2	3	3	3	3	2	2	2	2	2	6
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10		1	1	2	2	7	4	4	0	0	-1	-1	-2					10
11		-2	1	2	2	0	0	0	0	0	-1	-1	-2					11
12		-2	-2	-2	-1	-1	0	0	0	-1	-1	-2						12
13			-2	-2	-2	-1	-1	0	-1	-1	-2							13
14				-2	-2			-2	-2	-2	-2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

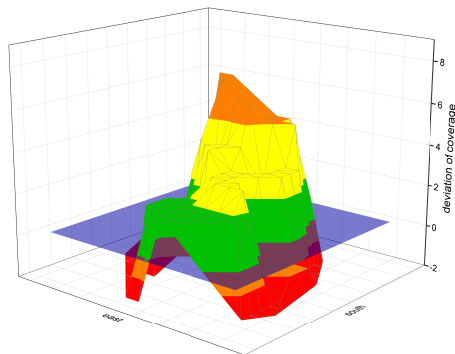
status quo

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													-1					1
2							-2	1		3	0	0	-1	-1	-1			2
3						-2	1	1	3	3	3	0	2	1				3
4			-2	-2	-2	1	1	2	2	3	3	5	2	2	0			4
5			0	0	0	1	1	2	2	2	5	5	2	1	0	0		5
6	-2	0	0	0	0	2	0	1	2	2	4	4	1	1	0	0	0	6
7	0	0	0	0	0	0	3	3	3	3	4	1	1	0	0	0	0	7
8	0	0	0	0	1	0	2	3	3	4	1	1	0	0	0	0	-2	8
9	0	0	0	1	1	3	3	0	1	1	0	0	0	0	0	-2	-2	9
10		1	2	3	2	3	1	1	1	1	0	0	-2					10
11		0	2	3	3	2	1	1	1	1	0	0	-2					11
12		0	0	0	1	1	2	1	1	0	0	-2						12
13			0	0	0	1	0	1	0	0	-2							13
14				0	0			-2	-2	-2	-2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

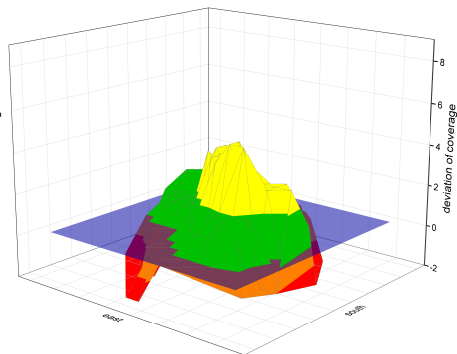
max empirically required coverage

# Coverage level

Difference to empirically required coverage (period 8-12 a.m.)



current allocation



optimal allocation

# Integration in a Decision Support Tool

## Main Menu

The screenshot shows the main menu of the SP<sup>2</sup> (Strategische Planung von Ressourcen im Rettungswesen) software. The window title is "Hauptübersicht". The interface features a dark blue header with the SP<sup>2</sup> logo and the text "Strategische Planung von Ressourcen im Rettungswesen". To the right, there are logos for "Stiftung Zukunft NRW" and "RUHR-UNIVERSITÄT BOCHUM" with "RUB OR" and "PROF. DR. BRIGITTE WERNERS". Below the header is a navigation bar with four tabs: "Optimierung", "Simulation", "Prognose", and "Ergebnisanalyse". The main content area contains several buttons: "Einsatzdaten laden" at the top center; four large buttons in a row: "Simulation" (with a green play button icon), "Optimierung" (with "max" and a summation symbol  $\Sigma$ ), "Prognose" (with a red line graph icon), and "Ergebnisanalyse" (with a bar chart icon); "Konfiguration" and "Session" buttons below the row; and a footer area with two text boxes: "Ruhr-Universität Bochum, Operations Research, Lehrstuhl für Betriebswirtschaftslehre, insbes. Unternehmensforschung und Rechnungswesen, Entwickler." and "Beta Version 2.1a, Datum 05.05.2014". A "Beenden" button is located in the bottom right corner.



# Integration in a Decision Support Tool

## Input Parameters

Optimierung

**SPR<sup>2</sup>** Strategische Planung von Ressourcen im Rettungsdienst

RUHR-UNIVERSITÄT BOCHUM  
Stiftung Zukunft NRW  
PROF. DR. BRIGITTE WERNERS

Optimierung    Simulation    Prognose    Ergebnisanalyse

Anzahl Zeitperioden

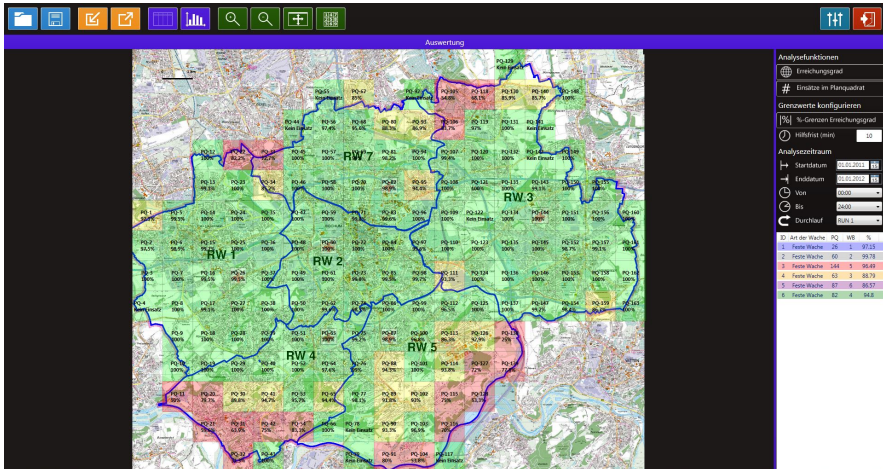
in Zeitperiode  plazierte Anzahl an Fahrzeugen

Einsatzdaten  historisch  prognostiziert

mobile Wachen  Umplatzierung  Belegungsvorgaben

# Integration in a Decision Support Tool

Extensive analyses – real-world performance measures



# Conclusions and outlook

## Improvements & further research

- ▶ Quality improvements:
  - ▶ Consideration of flexible ambulance stations is recommended
  - ▶ More suitable coverage according to empirical demands
- ▶ Efficiency of resource utilization:
  - ▶ Reduced number of ambulances
  - ▶ Constant quality with cost reduction
  - ▶ Higher quality with equal costs
- ▶ Integration of uncertainties (demand, driving speed)
  - ▶ Robust Uncertain Set Covering Problem (considers uncertainty in driving speed)
- ▶ Evaluation of the solution with a discrete event simulation

# Contact

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## Literature — Selection

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- ▶ Degel, Dirk; Wiesche, Lara; Rachuba, Sebastian; Werners, Brigitte (2014): Time-dependent ambulance allocation considering data-driven empirically required coverage, in: *Health Care Management Science* (online first).
- ▶ Gendreau, M.; Laporte, G.; Semet, F. (2001): A dynamic model and parallel Tabu search heuristic for real-time ambulance relocation, in: *Parallel Computing*, Vol. 27, No. 12, 2001, S. 1641-1653.
- ▶ Kritzinger, Stefanie et al. (2011): Variable neighborhood search for the time-dependent vehicle routing problem with soft time windows, in: Coello Coello, C.(Hrsg.): *Learning and Intelligent Optimization*. 5th International Conference, LION 5, Berlin 2011, S. 61-75.
- ▶ Schmid, V.; Doerner, K. (2010): Ambulance location and relocation problems with time-dependent travel times, in: *European Journal of Operational Research*, Vol. 207, No. 3, 2010, S. 1293-1303.
- ▶ Statistisches Bundesamt (2013): *Bevoelkerungsvorausberechnung*.
- ▶ Zarandi, Fazel M.H.; Davan, S.; Sisakht, Haddad (2011): The large scale maximal covering location problem, in: *Scientia Iranica*, Vol. 18, No. 6, 2011, S. 1564-1570.

# Optimization model

## Objectives

- ▶ maximize the demand which get the required coverage

$$\max \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} d_{it} x_{it}^{e(it)}$$

- ▶ avoid relocations

$$\min \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} u_{ijt}$$

- ▶ avoid the use of flexible EMS stations

$$\min \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{F}} y_{jt}$$

# Optimization model

- ▶ location/covering constraints

$$\sum_{j \in \mathcal{N}_{it}^2} y_{jt} \geq 1 \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

$$\sum_{j \in \mathcal{N}_{it}^1} y_{jt} \geq \sum_{k=1}^p x_{it}^k \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

$$x_{it}^{(k-1)} \geq x_{it}^k \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, k = 2, \dots, p$$

$$\sum_{j \in \mathcal{J}} y_{jt} \leq p_t \quad \forall t \in \mathcal{T}$$

# Optimization model

## ► relocation constraints

$$\begin{aligned}y_{Dt} &= p - p_t && \forall t \in \mathcal{T} \\y_{jt} + \sum_{i \in \mathcal{J} \cup \{D\}} u_{ij(t+1)} - \sum_{i \in \mathcal{J} \cup \{D\}} u_{ji(t+1)} &= y_{j(t+1)} && \forall j \in \mathcal{J} \cup \{D\}, \forall t \in \mathcal{T} \setminus \{T\} \\y_{jT} + \sum_{i \in \mathcal{J} \cup \{D\}} u_{ij1} - \sum_{i \in \mathcal{J} \cup \{D\}} u_{ji1} &= y_{j1} && \forall j \in \mathcal{J} \cup \{D\} \\ \sum_{i \in \mathcal{J}} u_{iDt} &\leq |p_{(t-1)} - p_t| && \forall t \in \mathcal{T} \setminus \{1\} \\ \sum_{j \in \mathcal{J}} u_{Djt} &\leq |p_{(t-1)} - p_t| && \forall t \in \mathcal{T} \setminus \{1\} \\ \sum_{i \in \mathcal{J}} u_{iD1} &\leq |p_T - p_1| \\ \sum_{j \in \mathcal{J}} u_{Dj1} &\leq |p_T - p_1|\end{aligned}$$



# Optimization model

## ► demand constraints

$$\sum_{i \in \mathcal{I}} d_{it} x_{it}^1 \geq \alpha \sum_{i \in \mathcal{I}} d_{it} \quad \forall t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{I}} d_{it} \leq c \sum_{j \in \mathcal{J}} y_{jt} \quad \forall t \in \mathcal{T}$$

## ► site capacity constraints

$$y_{jt} \leq \rho_j \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$$

$$y_{jT} = 0 \quad \forall j \in \mathcal{J}$$

## ► variable domain

$$x_{it}^k \in \{0, 1\} \quad \forall i \in \mathcal{I}, k = 1, \dots, p, \forall t \in \mathcal{T}$$

$$y_{jt} \in \{0, 1, \dots, p\} \subset \mathbb{N}_0 \quad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$$

$$u_{ijt} \in \{0, 1, \dots, p\} \subset \mathbb{N}_0 \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$$